Direct proof, indirect proof, and proofs of equivalence

* There are 3 types of proofs; Direct proofs, Indirect proofs, and proofs of equivalence.
* In Direct proof, we start with the assumption that the premise (p) is true. Then prove that q is true because of p.
* E.g: - proving that if x = 2, then *x^*2+3*x*=10.
* In an indirect proof, we prove the contrapositive, which is equivalent to proving the original statement.
* For p -> q, we prove ¬q→¬p. E.g: - Proving that if x^2+3x=10, then x≠3.
* In proof of equivalence, it involves proving both direction: *p*→*q* and *q*→*p*.
* *p*↔*q* is equivalent to (*p*→*q*) ∧ (*q*→*p*). E.g: - Proving that *n* is even if and only if *n^*2 is even.
* Its mainly useful in reasoning, creativity, abstract or spatial thinking, critical thinking, problem-solving ability, and even effective communication skills.

Proof by contradiction, cases, and counter examples

* There are 3 more types of proofs; Proof by Contradiction, proof by cases, and proof by counter example.
* In proof by contradiction, to prove a proposition p, one assumes its negation (¬p) and then derives a contradiction.
* Example: Proving there is no largest integer or that √2 is irrational.
* The proof establishes the truth of p by showing that ¬p leads to an impossibility.
* In proof by cases, instead of proving a direct implication p→q, proof by cases breaks down the proof into different cases, proving each individually.
* Example: Proving that for any integer n, n^2+n is even by considering cases based on whether n is even or odd.
* Cases cover all possibilities, ensuring a comprehensive proof.
* In proof by counter example, to disprove a universal statement, a single counterexample is sufficient.
* Example: Disproving statements like "For all integers n, 3n^2 + 1 is even" or "For all real numbers x, x^2 ≥ x using a counterexample.
* Counter examples provide a concrete instance where the universal statement fails.
* It is mainly useful in reasoning, creativity, abstract or spatial thinking, critical thinking, problem-solving ability, and even effective communication skills.

References

1. *Math in daily life*. (n.d.). Cuemath. <https://www.cuemath.com/learn/math-in-daily-life/>